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ABSTRACT

This status study provides evidence supporting the feasibility of teaching certain probability concepts to fifth and sixth grade students. The study employed an interview technique to analyze the status of three basic concepts of probability--sample space, probability of an event, choosing the most likely of two events. The effects of visual aids on the child's ability to solve problems involving these concepts were also measured. Socioeconomic, grade, and I.Q. differences were explored in relation to visual aids and the three concepts. The study indicated that the status of these concepts is related to I.Q. socioeconomic background, and to a lesser degree, grade differences. All differences were in the expected direction. The children had learned a significant amount about the three concepts without formal instruction. The children from a middle-class area were more successful at the verbal level than the children from the low socioeconomic area. The study showed that none of the students knew initially to assign a probability number to the chances of an event occurring. The study indicates that the counting of sample points is a necessary, but not a sufficient, condition for assigning a probability to an event. (Author/FL)

AN EXPLORATORY STUDY OF THE INTERACTION OF THREE ELEMENTARY CONCEPTS OF PROBABILITY WITH STIMULI, SOCIOECONOMIC, AND IQ DIFFERENCES

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Technical Report No. 114

AN EXPLORATORY STUDY OF THE INTERACTION OF THREE ELEMENTARY CONCEPTS OF PROBABILITY WITH STIMULI, SOCIOECONOMIC, GRADE, AND IQ DIFFERENCES

by Jack L. Shepler

Report from the Project on
Analysis of Mathematics Instruction

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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This status study, conducted in May 1968, provides evidence supporting the feasibility of teaching certain probability concepts to at least 30 Fifth and Sixth Grade students from Racine, Wisconsin. The study employed an interview technique to analyze the status of three basic concepts of probability (sample space, probability of an event, choosing the most (equally) likely of two events) and measured the effects of visual aids on the child's ability to solve problems involving these concepts.

Socio economic, grade, and IQ differences were explored in relation to visual aids and the three concepts. The study indicated that the status of these concepts is related to IQ, socioeconomic background, and to a lesser degree, grade differences. All differences were in the expected direction. The children had learned a significant amount concerning the three concepts without formal instruction. However, the children from a middle-class area were more successful at the verbal level than the children from the low socio-economic area. Many times these latter children needed the picture and model of the problem with verbal hints in order to answer an item correctly. The study showed that none of the students knew initially how to assign a probability number to the chances of an event occurring. The study indicates that the counting of sample points is a necessary, but not a sufficient, condition for assigning a probability to an event.

I

BACKGROUND AND RELATED RESEARCH

INTRODUCTION

The world we live in is a world of uncertainty. There are very few absolute laws which can be used to predict exact, precise outcomes. Since this is true, our modern technological society has increasingly called upon the mathematics of the probable—probability as a model to foretell outcomes in this unpredictable world. Daily, we hear statements such as: there's a 50% chance of rain today; the Packers are a 5-3 bet to win the professional football championship; McCarthy has a 7-1 chance of being nominated for president, etc.

Because it is important to understand and be able to use statements such as the above, probability has become an important subject in our schools. Until recently the study of probability was limited to the high school and college. Inhelder states in Bruner's The Process of Education, that this

> ...may be due to the widespread belief that the understanding of random phenomena depends on the learner's grasp of the meaning of the rarity or commonness of an event. Our research indicates that the understanding of random phenomena requires, rather, the use of certain concrete logical operations well within the grasp of the young child provided these operations are free of awkward mathematical expression. (p. 45)

Today one will find many junior high mathematics texts containing probability units. A later development has been to include probability units in some of the new mathematics programs for the elementary school. Page (1959) and Smith (1963) make suggestions for activities to include in the elementary school with regard to probability. The Cambridge Conference on School Mathematics (1963) also lists specific activities that might be appropriate for Grades 3, 4, 5, and 6.

Most of these people, including the authors of elementary textbooks, assume the child has some intuitive notions about certain basic concepts of probability. Some of these basic concepts are: identifying all of the outcomes in a sample space, assigning probabilities to simple events, and recognizing when events are equally likely. These are the basic concepts with which this study is concerned.

SUMMARY OF RELATED RESEARCH

There are a number of studies which shed some light on the question as to what children may have learned from their environment concerning certain concepts of probability without formal instruction and when this takes place. Of the status studies concerning probability and statistical concepts possessed by children, the most diverse and influential have been those conducted by Piaget and Inhelder and summarized by Flavell (1963). Through a series of ingenious tasks given to children those authors observed three stages in their development of the ideas of chance.

They also have approximated age intervals for the subjects they observed for these stages of development.

The first stage (approximately up to age 7) is identified as the preoperational stage. Flavell (1963) states:

> In order to identify a set of phenomena as 'chance events' one first has to identify a set of phenomena which are not chance events, a nonchance ground against which chance can emerge as figure. (pp. 341-342)

The child in this stage lacks the intellectual operations necessary to recognize events which

are certain, much less those which are uncertain. There is a generalized undifferentiation between chance and nonchance, between the possible and the necessary during this development period. In this stage the child would show little or no consternation in getting 20 heads in 20 flips of a coin. They are unable to grasp the extreme unlikelihood of such an event.

The second stage of development, the concrete operational stage, occurs when the child recognizes events he can know and those he can only guess. He clearly recognizes chance events when he encounters them. However, when faced with predicting an outcome, he will do this in terms of the absolute number of favorable or unfavorable outcomes. He typically cannot list all the possible outcomes for a more complex event.

In the third stage, the period of formal operations (age 11 and up), the child is described as being able to think in terms of combinations, permutations, and proportions. Thus he now possesses the intellectual tools to handle many chance events adequately where previously he could only recognize them. In the previous stage, the child is able to quantify simple events but makes his decisions in terms of absolute differences. The child at the formal operational stage is able to quantify the event and make correct decisions in terms of proportions. It is also recognized that some individuals progress only incompletely through all three stages and thus the concepts of chance may only be partially acquired.

Some studies have been conducted to test Piaget's and Inhelder's conclusions. Of these, only a few will be briefly cited.

Davies (1965) quotes the studies of Pire (1958) and Yost et al. (1962) as reporting a relationship between mental age and understanding of probability. In the same article, Davies confirms Piaget's notion that the concept of probability is a developmental phenomenon. However, she felt Piaget's interviews depended highly on verbal ability. She concluded that the non-verbal behavior reflecting event probability appears earlier than the verbalization of the concept of the probability or of its applications to tasks. She also concluded there do not seem to be sex differences.

The studies of Davies (1965), Goldberg (1966), and Yost, et al. (1962) also seem to show that young children, 4-6 years of age, do demonstrate some understanding of probability under appropriate conditions, contrary to Piaget's and Inhelder's results.

Leffin (1968) conducted a status study of three concepts of probability possessed by 528 children in the Fourth, Fifth, Sixth, and Seventh Grades at Wausau, Wisconsin. The three concepts investigated were the points of a sample space, the probability of a simple event, and the quantification of probabilities (the latter measured by the students' ability to choose the most likely (equally likely) event of two events). Three tests, one for each of the three concepts, were constructed and administered to the subjects. From the multivariate analysis of the data, the following results were pertinent to this study. There were significant results with respect to IQ and grade level, ordered in the expected direction. The discrimination among grades was due primarily to Test I, sample space. Children in Grades Six and Seven performed better on all three tests than those in Fourth and Fifth Grades and demonstrated a better understanding of sample space, particularly concerning the associated notions of unordered events and ordered events. From a correlational study of the data from the tests it appeared that one could not be certain that a child understood the idea of sample space because he answered correctly some questions on a paper-and-pencil test about the probability of simple events. The three tests were correlated for each grade with the correlation coefficients ranging from .40 to .58. The tests were interdependent as would be expected. There was a pattern of very low inter-item correlations between pairs on corresponding items (e.g., a problem asking the student to list the points of a sample space on Test I and the parallel item on Test II asking the student to specify the probability of a simple event).

Leffin (1968) states:

> The most significant outcome of this study is that the children demonstrated that they had acquired considerable knowledge about the three concepts of probability under investigation and could apply these concepts in a variety of situations. (p. 192)

Doherty (1965) studied the levels of four concepts of probability (sample space, probability of a simple event, probability of the union of mutually exclusive events in a sample space, difference between mutually independent events and mutually exclusive events) possessed by (54 Ss) in Grades 4, 5, and 6. She reported that:

(1) Children have acquired considerable familiarity and ability with the four concepts as measured by her test.

(2) There is no significant differences in the level of difficulty among the four concepts.

(3) There is no sex difference.

(4) There are significant differences in performance in children with regard to mental age, with respect to the rating of the child by the teacher, and with respect to average achievement levels, with performance increasing as the forementioned ratings increased.

She recommended that children be given formal instruction in probability in the intermediate grades and that a study be undertaken to discover which methods and materials produce the best results in learning probability.

A status study by Leake (1962) showed that junior high school children have gained considerable knowledge of three basic concepts of probability. Leake constructed a 30-item test to measure the concepts of (1) sample space, (2) the probability of a simple event, and (3) the probability of the union of non-overlapping events in the sample space.

That test was administered to 72 children in Grades 7, 8, and 9 from upper middle-class neighborhoods in Cincinnati, Ohio. Only children classified in achievement as average and superior academic students were used in the study which showed significant grade differences ordered in favor of the higher grade(s), achievement differences in favor of the superior student, and differences as to the students' ability to solve the three concepts. Concepts 1, 2, and 3, in that order, ranked from least difficult to most difficult. There also was a far greater correlation between mental age and scores than between chronological age and scores.

There are a host of research studies dealing with probability learning experiments. These studies are concerned with the application of subjective judgments in risk situations or learning experiences which include some aspects of probability as it is applied to a specific task. These studies have little evidence to offer concerning the status of a concept, constructing a curriculum in probability and statistics, or the behaviors students would exhibit after taking such a course.

The repeated themes running through the research cited here seem to be that children are learning a significant amount concerning concepts of probability from their environment without formal instruction and that this is taking place at an earlier age than many educators expected. Some studies also show that mental age, achievement, and grade level may be helpful in predicting the status of these concepts.

However, no study has explored the effects of different audio-visual stimuli on the status of children's knowledge of certain concepts of probability. No study has attempted to compare the status of students' knowledge of probability concepts from a school in a low socio economic area in contrast to a school in a middle socio-economic area.

Leffin (1968) states "..., one can not be certain that a child who gives a correct response to a question about the probability of an event actually recognizes all the elements of the sample space which contains the event" (p. 194). With respect to choosing between two urns, he states:

> It is not clear from the children's written responses how they actually think about such items or how they decide which answer is correct. Further study with the type of item used in Test III, using an interview technique, could provide valuable information about children's understanding of this concept as well as a deep insight into their understanding of sample space and probability of an event (p. 195).

This study is designed to undertake Leffin's suggestions. Also the relationship between sample space, probability, and choosing between two urns has not been sufficiently explored. Several studies report grade and IQ differences ordered in the expected direction. This study was designed to explore these differences using an interview technique and examining the relationship of the grade and IQ variables in relation to socioeconomic differences and different visual stimuli.

II
PROBLEM AND PROCEDURE

PURPOSE OF THE STUDY

The objectives of this exploratory study were to analyze:

(1) the status of three basic concepts of mathematical probability with children of different IQs in Grades 5 and 6.

(2) the effects of the presence or absence of visual aids on the child's ability to solve problems involving these three basic concepts.

(3) the differences of children's conception of these concepts when the school and socioeconomic environment are quite different.

The concepts included in this study were:

(1) the points of a finite sample space (e.g., the outcomes possible in throwing a die are 1, 2, 3, 4, 5, or 6).

(2) the probability of an event for a finite sample space (e.g., the probability of rolling a two with a die is 1/6 (1 out of 6).

(3) the most likely (equally likely) occurrence of an event in one trial where the event is a subset or each of two distinct sample spaces (e.g., selecting which of two urns affords the best chance of picking a red chip in one draw if both urns contain an unequal number of red and blue chips).

This study is an outgrowth of Leffin's (1968) status study and incorporates six items (Items 2, 3, 4, 5, 7, 8) similar to items in his test (pp. 202-222). The three concepts included in Leffin's study are included here; however, the probability of a simple event has been broadened to include the probability of an event (simple or compound). There is also an overlap in the grades considered. Leffin included Grades 4-7 while this study centers on Grades 5 and 6. By employing an interview technique rather than a written test it was anticipated that further information could be gained concerning these concepts. Also the variables of socioeconomic status and the effects of different visual stimuli were incorporated in the interview.

HYPOTHESES TO BE CONSIDERED

The following are the hypotheses considered for this study:

1. The level of achievement recorded by students in Grade 6 is higher than that for students in Grade 5.
2. The level of achievement of a school in a middle socioeconomic area is higher than that of a school in a low socioeconomic area.
3. The level of achievement of students with higher IQ is greater than students with lower IQ.
4. Students in Grade 6 score higher on the more abstract levels of stimuli than students in Grade 5.
5. Students in a school in a middle socioeconomic area score higher on the more abstract levels of stimuli than students in a low socioeconomic area.
6. Students having higher IQs score higher on the more abstract levels of stimuli than students having lower IQs.
7. The ability to list sample points is related to the ability to find the probability of an event.
8. The ability to list sample points is related to the ability to choose when two events are equally or unequally possible.
9. The ability to find that the probability of an event is related to the ability to choose when two events are equally or unequally possible.

Descriptive comparisons using percentages and ratios are employed to analyze the data. Thus one can see if the data support or produce evidence contrary to the stated hypotheses.

PROCEDURE

To analyze the three-fold purpose, a structured interview involving 12 problems was constructed. Two small pilot studies were conducted and from these results, further modifications of the interview were made. An experienced mathematics coordinator and the author conducted the interviews. A day was spent in attempting to standardize the interview procedures of the two interviewers prior to the study. One should note that for the study, random assignment of subjects to interviewers was not possible.

With regard to the problems used, three questions (1a, 2, 3) involving the concept of listing points in a sample space (one dimensional, ordered pairs, and unordered pairs of points) were included. Seven questions (1b-f, 7, 8) were employed to measure the ability of the child to assign a probability to an event. Three questions (4, 5, 6) were included to measure the child's ability to recognize the most likely occurrence of an event. (See pp. 15-17 for examples of the above).

The apparatus used in the interview were: two sets of cards containing the statements of the problems—one for the child and one for the interviewer; a set of pictures of the problems; two transparent containers; poker chips (black, red, white, blue); and a board with two spinners. The interviewer's cards listed the hints to be given for the appropriate stimulus levels. The interviewer explained the purpose of the experiment, read aloud each problem, presented the appropriate stimulus in the correct order—Verbal (V), Verbal Hint (VH), Picture (P), and Model (M)—and recorded the student's response (see Appendix B). A sheet of paper and a pencil were given to the student to record this answer (see Appendix B).

It is assumed in this study that the use of visual stimuli aids in problem solving increases the student's ability to solve that problem and that the presentation is ordered from least to most helpful. Theoretical support for this order derives from Dale's (1950) postulation of a "cone of experience," and Bruner's (1966) "enactive, iconic, and symbolic" stages of cognitive development. For this set order, the study hoped to find which students needed what types of visual aids in order to answer the questions successfully.

The procedure for each question was for the interviewer to first read the question. If the child could not give a correct response accompanied by a correct reason to the question "why?," when appropriate, a verbal hint as to the intent of the question was given. If the child was then unsuccessful, a picture of the problem accompanied by the interviewer's rereading the question was employed. If the child was still unsuccessful, a model of the problem was given to him to use. A number of hints could then be employed by the interviewer until the subject either responded correctly or the hints were exhausted. At this point or at any point at which the child was successful in giving the correct response and reason, the interviewer would proceed to the next problem. A sheet was given to the interviewer to help in recording the reason given (see Appendix B). There was one modification of this procedure. If the student was able to give a correct answer at the verbal level, a picture was shown him to see if he would change his answer after seeing the picture.

It should be noted that classifying reasons to the question "why?" was not as easy as had been anticipated. This was particularly true of some children from the low socioeconomic area who had serious verbal deficiencies. At times their reasons were nonsensical or vague. Thus it was decided that the interviewer would conclude whether the reason was correct or try to write it down so that a decision could be made after the interview.

THE SAMPLE

The study was done in cooperation with the Racine Public Schools, Racine, Wisconsin, in May 1968. Since this study is exploratory in nature, it was decided to allow the principals and the teachers of their respective schools to choose students according to IQ levels. The Fifth and Sixth Grade students were selected with regard to three levels of IQ, L (105 and below), M (106-125), and H (126 and above). To obtain information concerning socioeconomic and cultural differences, two different schools were used. Elementary School A was in a low socioeconomic section of the city and was 65% black. (Approximately the same percentage of Negroes was included in the sample.) Elementary School B was located in a newer, middle socioeconomic area of the city and was all white.

SAMPLE (N=30)

School A	IQ H	IQ M	IQ L	Total
Grade 5	3	2	3	8
Grade 6	2	3	3	8
School B				
Grade 5	2	2	2	6
Grade 6	3	3	2	8
	10	10	10	30

Total Number of Students from School A and School B

Figure 1. Description of Sample by School, Grade, and IQ

III
RESULTS AND CONCLUSIONS

RESULTS

An item-by-item description will be discussed followed by a summary of the results across different variables to seek supportive evidence for the stated hypotheses.

Items 1(a-e) will be considered separately from Items 2 through 8 since the former items are concerned with the same sample space and the picture and model are the same. Thus if a subject missed Item 1 b at the Verbal (V) and Verbal Hint (VH) Levels, he is given the picture related to the problem. He then is at the Picture Level of stimulus for Items 1(c-e). A similar situation is true for the Model Level also. Hence the various levels of visual stimuli can only be analyzed for Item 1 b and the overall trend across Items 1 (b-e). Item 1 a is considered a warm-up and all 30 Ss were able to answer the item correctly. It should be noted however, that two students from School A (Grade 5 and Grade 6) required a picture to answer the question. An example of how to interpret the tables for each item follows Item 1 b.

Items 1 (a-f)

Item 1.

In this experiment a box contains six chips —three white, two red, and one black. You pick one chip from the box without looking.

a. What are all the different colors it would be possible to obtain for this experiment?

b. What is your chance of getting a white chip?

c. What is your chance of getting a black chip?

d. What is your chance of getting a red or white chip?

e. Imagine that the first chip you pick is a black chip. You do not put this chip back into the box. If you pick again, now what is your chance in picking a black chip? a red or a white chip?

Hint

1.V What I mean is an answer like 1/5 or 3/11.

2.V For example, there are 2 red and 6 chips altogether so your chances are 2/6.

3.M What do you think of your chances of winning? (Are they good, bad, or neither?)

4.M How many chips altogether?

5.M How many (white, black, or red) chips are there?

(This item is designed to measure the listing points of a one-dimensional sample space (1a.) and the probability of an event [1(b-f)].)

Item 1 b.

What is your chance of getting a white chip? (This item is designed to measure the probability of a compound event.)

In order to interpret the table consider School A (Grade 5). At the Verbal Level (V) looking at the high IQ group (H), zero out of

Table 1
Data for Item 1 b

IQ	V (Verbal)			VH (Verbal Hint) Hint 1			VH (Verbal Hint) Hint 2			P (Picture)			M (Model)			Total Sum		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	0/3	0/2	0/3	2/3	1/2	1/3	1/1	0/1	0/2	0/0	0/1	0/2	3/3	1/2	1/3
School A (Grade 6)	0/2	0/3	0/3	1/2	0/3	0/3	0/1	1/3	1/3	1/1	1/2	0/2	0/0	1/1	1/2	2/2	3/3	2/3
School B (Grade 5)	0/2	0/2	0/2	1/2	2/2	0/2	1/1	0/0	2/2	0/0	1/2	0/1	0/0	1/1	1/1	2/2	2/2	2/2
School B (Grade 6)	0/3	0/3	0/2	2/3	1/3	0/2	1/1	0/2	1/2	0/1	1/2	0/5	0/0	2/3	2/5	3/3	3/3	2/2
Total Sum (Ratio form)	0/10	0/10	0/10	4/10	3/10	0/10	4/6	2/7	5/10	2/2	2/5	0/5	0/0	2/3	2/5	10/10	9/10	7/10

three subjects (0/3) answered the question correctly; at the Verbal Hint 2 Level (VH 2), 2/3 (two out of three) of group H answered correctly; and at the Picture Level, the one person in this group who has not responded correctly answers the item at Picture level (P) of stimulus. At the Model Level (M), 0/0 (zero subjects out of zero subjects), reflected the fact that all the persons had already answered the item correctly at the preceding levels of stimuli.

This item is one of the most interesting in the study in that it showed that no student (0/30) was able to answer the question correctly without some aid. At the Verbal Level this item elicited responses such as there is "a good chance" (8/30), "a pretty good chance" (3/30), a number cited (4/30) (The numbers cited were 70%, 3 chances, 1/5, and 6/10.), "more white" (1/30), and no reply (14/30). Since no one answered the question correctly, it was concluded that for these children the quantitative concept of chance did not exist at the Verbal Level. However, when the first verbal hint was given, ("What I mean is an answer like 1/5 or 3/11."), seven (7/30) children were able to respond correctly. Eleven (11/23) more responded correctly when given the second verbal hint (for example, there are 2 red and 6 chips altogether so your chances are 2/6).

With respect to schools 12/16 from School A answered the item correctly while 14/14 from School B responded correctly. With respect to IQ at the VH 1 level, 7/20 of the high (H) and medium (M) IQ groups answered correctly in contrast to 0/10 from the low IQ group (L). A similar result held for the Picture Level (4/7 for the H and M groups, 0/5 for the low IQ group). At the Model Level with hints 2/3 for the H and M groups answered correctly while 2/5 of the L group were successful. With respect to grade level, 11/14 from Grade 5 and 15/16 from Grade 6 were successful, with those missing the item being from School A.

One should note that 8 Ss were functioning at the M level for Items 1(c-e), 6 Ss from School A and 2 Ss from School B and 4 Ss (3 S - A, 1 S - B) were at the Picture Level for 1 c. That left 7/16 from A and 11/14 from B at the VH Level for Item 1 c.

Item 1 c.

What is your chance of getting a black chip? (This item is designed to measure the probability of a simple event.)

Table 2
Data for Item 1 c

LEVELS OF STIMULI

IQ	VH			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	2/3	1/2	1/3	1/1	0/1	0/2	0/0	1/1	2/2	3/3	2/2	3/3
School A (Grade 6)	1/2	1/3	1/3	1/1	1/2	0/2	0/0	0/1	1/2	2/2	2/3	2/3
School B (Grade 5)	2/2	2/2	2/2	0/0	0/0	0/0	0/0	0/0	0/0	2/2	2/2	2/2
School B (Grade 6)	3/3	1/3	1/2	0/0	1/2	0/1	0/0	1/1	1/1	3/3	3/3	2/2
Total	8/10	5/10	5/10	2/2	2/5	0/5	0/0	2/3	4/5	10/10	9/10	9/10

With respect to schools, 14/16 from School A and 14/14 from School B answered the item correctly. There was little difference with respect to IQ. (See Table 2.)

With respect to grade 14/14 from Grade 5 and 14/16 from Grade 6 answered the item correctly. One should note that levels of stimuli are almost meaningless for Items 1 (c-e) individually since one is bound in a stimulus level according to his performance on the previous item.

Item 1 d.

What are your chances of getting a red or white chip? (This item is designed to measure the probability of the union of two disjoint events.)

With respect to schools 8/16—A and 14/14—B answered the item correctly. The children from School B were able to focus on the number of objects which possessed a dissimilar

Table 3
Data for Item 1 d

LEVELS OF STIMULI

IQ	VH			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	1/3	2/3	0/2	0/2	1/1	0/2	0/2	3/3	0/2	1/3
School A (Grade 6)	1/2	1/3	1/3	1/1	0/2	0/2	0/0	0/2	/2	2/2	1/3	1/3
School B (Grade 5)	2/2	2/2	2/2	0/0	0/0	0/0	0/0	0/0	0/0	2/2	2/2	2/2
School B (Grade 6)	3/3	1/3	1/2	0/0	1/2	0/1	0/0	1/1	1/1	3/3	3/3	2/2
Total	6/10	4/10	5/10	3/4	1/6	0/5	1/1	1/5	1/5	10/10	6/10	6/10

characteristic. The children from School A gave answers such as "3/6 or 2/6," "2/6" (since there are 2 colors), and "5 out of 1" (possibly the odds?). With respect to IQ, 9/10 from H, 5/10 from M, and 5/10 from L answered correctly at either the VH or P levels. Overall 10/10—H, 6/10—M, and 6/10—L responded correctly. (See Table 3.)

With respect to grades 10/14—G. 5 and 12/16—G. 6 responded correctly with those missing the item being from School B.

Item 1 e.

> Imagine that the first chip you pick is a <u>black</u> chip. <u>You</u> <u>do</u> <u>not</u> put this chip back into the box. If you pick again, now what is your chance in picking a black chip? (This item is designed to measure the concept of sampling without replacement and the probability of the impossible event.)

This item produced little differences, mainly because 29/30 answered "impossible," "none," or 0/5. All of these were accepted as being correct. If the author was to do the study again he would try to probe to see if children would assign the number 0 to the impossible event. (See Table 4.)

Item 1 f.

> With respect to 1 e., now what is your chance in picking a <u>red</u> or a <u>white</u> chip? (This item is designed to measure the probability of two disjoint events in relation to sampling without replacement. This results in the sure event which has probability 1.)

This item produced school differences with 9/16 from A and 12/14 from B answering the item correctly. Also with respect to IQ there was a slight favoring of H (8/10) and M (8/10) over L (5/10). (See Table 5.)

SUMMARY (Items I (b-e))

Overall, for Items 1 (b-e), there were very little grade differences in the percentages responding correctly with regard to the level of stimulus (V, VH, P, M). (See Table 6.) However, there was a large school difference at the Verbal Level (School A, 19%; School B, 59%). This difference was understandable since many of the students in School A were less advanced in their reading ability than the children in School B. There seemed to be a greater need for the repeated reinforcement of the pictures and of the models and clues for the children

Table 4
Data for Item 1 e

LEVELS OF STIMULI

	VH			P			M			Total		
IQ	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	2/3	0/2	1/3	1/1	2/2	2/2	3/3	2/2	3/3
School A (Grade 6)	1/2	1/3	1/3	1/1	0/2	0/2	0/0	1/2	2/2	2/2	2/3	3/3
School B (Grade 5)	2/2	2/2	2/2	0/0	0/0	0/0	0/0	0/0	0/0	2/2	2/2	2/2
School B (Grade 6)	3/3	1/3	1/2	0/0	1/2	0/1	0/0	1/1	1/1	3/3	3/3	2/2
Total	6/10	4/10	4/10	3/4	1/6	1/6	1/1	4/5	5/5	10/10	9/10	10/10

Table 5
Data for Item 1 f

LEVELS OF STIMULI

IQ	VH			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	0/3	0/2	0/3	2/3	1/2	1/3	2/3	1/2	1/3
School A (Grade 6)	1/2	1/3	0/3	1/1	0/2	0/3	0/0	1/2	1/3	2/2	2/3	1/3
School B (Grade 5)	1/2	1/2	1/2	1/1	1/1	0/1	0/0	0/0	0/1	2/2	2/2	1/2
School B (Grade 6)	2/3	1/3	1/2	0/1	0/2	0/1	0/1	2/2	1/1	2/3	3/3	2/2
Total	4/10	3/10	2/10	2/6	1/7	0/8	2/4	4/6	3/8	8/10	8/10	5/10

Table 6

Percentages of Students Answering Items (1b-f) Correctly

Grade Comparison (%)

	V	VH	P	M	Total
Grade 5 (N=14)	36	14	13	20	83
Grade 6 (N=16)	38	10	14	24	85

School Comparison (%)

	V	VH	P	M	Total
School A (N=14)	19	09	18	27	73
School B (N=16)	59	16	09	16	97

IQ Comparison (%)

	V	VH	P	M	Total
H. (N=10)	46	16	24	10	96
M. (N=10)	32	10	14	26	82
L. (N=10)	32	10	10	30	74

from School A to respond correctly than for the children from School B.

Looking at IQ alone, the students in the high IQ group did better at the V, VH, and P level than the middle and low groups. The middle and low IQ groups performed approximately the same on Items 1-5. The overall achievement for the first five items was quite high: 84% (V-37%, VH-12%, P-14%, M-22%). With a minimum amount of instruction and guidance, most of the children in the study were able to find the probability of an event in a one-dimensional sample space and to explain correctly their response, and with respect to the applicable hypotheses (Hypotheses 1-6), all but Hypothesis 1 were supported by the data from Items 1(b-e).

Items 2-8

Items 2 through 8 will now be discussed individually and then collectively.

Item 2. In this experiment you have two spinners. The first spinner is colored half-white and half-red. The second spinner is colored half-blue and half-yellow.

You spin the arrow on <u>each</u> spinner. (If an arrow stops on a line you spin it again.)

Write the list of all the different sets of two colors it would be possible to obtain for this experiment. (Are these all of them or are there more?)

(This item is designed to measure the listing of ordered outcomes from a two-dimensional sample space.)

There was a school difference (9/16— A; 12/14— B) for this item. The V, VH, and P stimulus levels together show 5/10— H, 4/10 —M, and 1/10— L responding correctly. Thus these levels of stimuli seemed to be more helpful to the high and middle IQ groups. With respect to grades, the data (9/14— G. 5 and 12/16— G. 6) indicated very little grade difference. (See Table 7.)

Item 3. In this experiment a box contains four chips: there is a <u>red</u> chip, a <u>blue</u> chip, a <u>white</u> chip, and a <u>black</u> chip.

This time you pick <u>two chips at a time</u>.

What are all the different sets of two chips that it would be possible to

Table 7
Data for Item 2

IQ	LEVELS OF STIMULI	V			VH			P			M			Total		
		H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)		0/3	0/2	0/3	1/3	0/2	0/3	0/2	0/2	0/3	2/2	1/2	0/3	3/3	1/2	0/3
School A (Grade 6)		1/2	1/3	0/3	0/1	0/2	0/3	0/1	0/2	1/3	1/1	1/2	1/3	2/2	2/3	1/3
School B (Grade 5)		1/2	1/2	0/2	0/1	0/1	0/2	0/1	0/1	0/1	1/1	0/1	1/1	2/2	1/2	1/1
School B (Grade 6)		0/3	0/3	0/2	1/2	2/3	0/2	1/2	0/1	0/2	1/1	1/1	1/2	3/3	3/3	1/2
Total		2/10	2/10	0/10	2/8	2/8	1/10	1/6	0/6	0/9	5/5	3/6	3/9	10/10	7/10	4/10

obtain? (Are these all of them or are there more?)

(This item is designed to measure the listing of unordered outcomes from a two-dimensional sample space.)

There was an interesting school factor. All 14 students from School B answered Item 3 correctly while School A had 9/16 answering correctly. Of the nine who answered correctly from School A, five answered at the Model Level. Only one person from School B needed the Model Level to answer the question. For the seven who missed the item, the average person was able to write down four of the six combinations. The ability to consider all the possibilities is a characteristic of children whom Piaget classifies as being in the formal stage of intellectual development. The children from School A and School B were strikingly different: the former seemingly operated in terms of concrete objects, while the latter seemed to have no need for the concrete objects for this type of problem. (See Table 8.)

The IQ differences were particularly noticeable at the Verbal Level (7/10—H; 4/10—M; 1/10—L). Items 2 and 3 seemed to be related in that 4/30 answered Item 2 at the Verbal Level (V) while 12/30 answered Item 3 at the Verbal Level. The majority of students of higher IQ (7/10) seemed to see some relation between the two items. There was little overall grade difference (9/14—G. 5, 11/16—G. 6).

Item 4. Two boxes containing black and red chips are used to play this game. The first box contains three <u>black</u> chips and four <u>red</u> chips. The second box contains three <u>black</u> chips and five <u>red</u> chips.

To play this game you pick one chip from a box without looking. You win if you pick a <u>black</u> chip.

If you can play this game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

(This item is designed to measure the identifying of the most likely event of two events.)

There were little school differences (14/16—A; 13/14—B) or grade differences (11/14—G. 5, 16/16—G. 6). The slight grade difference favored the Sixth Grade. With respect to IQ, 9/20 in H and M groups in contrast to 1/10 in L answered correctly at the verbal level. (See Table 9.)

Table 8
Data for Item 3

LEVELS OF STIMULI

IQ	V			VH			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	1/3	0/2	0/3	0/2	0/2	0/3	0/2	0/2	0/3	2/2	0/2	1/3	3/3	0/2	1/3
School A (Grade 6)	1/2	0/3	1/3	1/1	0/3	0/2	0/0	0/3	0/2	0/0	2/3	0/2	2/2	2/3	1/3
School B (Grade 5)	2/2	1/2	0/2	0/0	0/1	0/2	0/0	1/1	1/2	0/0	0/0	1/1	2/2	2/2	2/2
School B (Grade 6)	3/3	3/3	0/2	0/0	0/0	0/2	0/0	0/0	2/2	0/0	0/0	0/0	3/3	3/3	2/2
Total	7/10	4/10	1/10	1/3	0/6	0/9	0/2	1/6	3/9	2/2	2/5	2/6	10/10	7/10	6/10

Table 9
Data for Item 4

LEVELS OF STIMULI

IQ	V			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	1/3	0/2	0/3	1/2	1/2	1/3	1/1	0/1	1/2	3/3	1/2	2/3
School A (Grade 6)	1/2	2/3	0/3	0/1	1/1	2/3	1/1	0/0	1/1	2/2	3/3	3/3
School B (Grade 5)	1/2	0/2	1/2	0/1	1/2	1/1	0/1	1/1	0/0	1/2	2/2	2/2
School B (Grade 6)	3/3	1/3	0/2	0/0	1/2	2/2	0/0	1/1	0/0	3/3	3/3	2/2
Total	6/10	3/10	1/10	1/4	4/7	6/9	2/3	2/3	2/3	9/10	9/10	9/10

The three students missing the item all said that there was no difference between the two boxes since they have the same number of black chips in each.

Item 5. Two boxes containing black and red chips are used to play this game. The first box contains one black chip and two red chips. The second box contains two black chips and four red chips.

To play this game you pick one chip from a box without looking. You win if you pick a black chip.

If you can play the game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

(This item is designed to measure the identifying of two equally likely events as being equally likely.)

This was the most difficult problem in the interview, with only 7/30 answering correctly. (See Table 10.) There is only one way to be able to make the correct decision (it doesn't make any difference) and give the correct reason why. The pupil must employ ratios. Most pupils chose either the second (7/30) because it had the most number of winners or the first (again 7/30) because it had the least number of losers. A few used a more sophisticated attack matching a winner with a loser and thus concluding the first box has one loser remaining while the second box has two losers left over. These strategies incorrectly employed absolute differences rather than ratios. Again, the most sophisticated were School B Sixth Graders. Although 5/8 missed the question, three of these five used ratios in making their incorrect decision, typically saying 1/3 is greater than 2/6. Of the Sixth Graders from School A only two used ratios to answer the question, one correctly and one incorrectly, whereas in School B, 6/8 used ratios. [It is interesting to note that only 3/14 Fifth Graders employed ratios.] Since there were few who answered the question correctly the differences were small, with respect to schools A—2/16; B—5/14); grade (3/15—G. 5; 4/16—G. 6); and IQ (4/10—H; 3/10—M; 0/10—L) It is interesting to note that no one in the low IQ group answered the item correctly.

Item 6. Two boxes containing black and red chips are used to play this game. The first box contains one black chip and three red chips. The second box contains two black chips and eight red chips.

To play this game you pick one chip from a box without looking. You win if you pick a black chip.

Table 10
Data for Item 5

LEVELS OF STIMULI

IQ	V			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	0/3	0/2	0/3	1/3	0/2	0/3	1/3	0/2	0/3
School A (Grade 6)	0/2	1/3	0/3	0/2	0/2	0/3	0/2	0/2	0/3	0/2	1/3	0/3
School B (Grade 5)	0/2	0/2	0/2	0/2	0/2	0/2	1/2	1/2	0/2	1/2	1/2	0/2
School B (Grade 6)	1/3	1/3	0/2	0/2	0/2	0/2	1/2	0/2	0/2	2/3	1/3	0/2
Total	1/10	2/10	0/10	0/9	0/8	0/10	3/9	1/8	0/10	4/10	3/10	0/10

If you can play this game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

(This item is designed to measure the identifying of the most likely event of two events.)

There were few interesting differences in performance for this item. The schools (A—13/16; B—12/14), IQ factor (9/10, 8/10, 8/10), and grades (12/14—G. 5, 13/16—G. 6) indicated little overall difference. There was an interesting reversal of the overall trend for IQ across items at the verbal level of School A in that the low group performed better than

Table 11
Data for Item 6

LEVELS OF STIMULI

IQ	V			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	2/3	0/2	2/3	0/1	1/2	0/1	1/1	1/1	0/1	3/3	2/2	2/3
School A (Grade 6)	0/2	2/3	3/3	0/2	0/1	0/0	1/2	0/1	0/0	1/2	2/3	3/3
School B (Grade 5)	2/2	2/2	1/2	0/0	0/0	0/1	0/0	0/0	0/1	2/2	2/2	1/2
School B (Grade 6)	3/3	2/3	2/2	0/0	0/1	0/0	0/0	0/1	0/0	3/3	2/3	2/2
Total	7/10	6/10	8/10	0/3	1/4	0/2	2/3	1/3	0/2	9/10	8/10	8/10

the other two groups (2/5 — H; 2/5 — M; 5/6 — L). (See Table 11.)

With regard to reasons, the majority (15/30) chose the first box since the second box has more red chips (i.e., more ways to lose); 5/30 incorrectly chose the second box, since there are more black chips in the second box than there are in the first box (i.e., more ways to win). Thus both of these groups centered on the absolute number of winners or losers. Of the correct responses, 10/30 employed ratios to arrive at the answer, 4 Ss using statements such as: "the first box since 2/5 is better than 3/9;" and 6 Ss using a combination of ratios and absolute differences.

Item 7. Two spinners are such that one is colored half-red and half-white and the other spinner is colored half-blue and half-yellow.

To play this game you spin the arrow on each of the spinners. (If an arrow stops on a line you spin it again.)

You win if one arrow points to red and the other arrow points to yellow.

You lose if the arrow stops in any other way.

If you only play this game once, what chance do you have of winning?

(This item is designed to measure the probability of a simple event in an ordered, two-dimensional sample space.)

This was a difficult item with only 13/30 correct answers. School differences favored B (4/16—A; 9/14—B) with the Sixth Grade from B accounting for this difference. The IQ differences favored the high and middle IQ groups (6/10—H; 5/10—M; 2/10—L). With respect to grades (4/14—A; 9/16—B) the differences were caused by the group from Grade 6, School B. Schools A - G. 5, A - G. 6, and B - G. 5 performed almost equally (2/8, 2/8, and 2/6, respectively), while B - G. 6 had 7/8 correct responses. (See Table 12.)

The majority of the wrong responses answered 2/4 (there are two colors that win and there are four colors altogether). This was true even though many of these had answered correctly in listing the sample points in Item 2. There was one correct answer (1/4) who gave the incorrect reason that there is one way to win and there are four colors. The incorrectness of this answer would not be caught by the typical written test.

Item 8. A box contains four chips, a red chip, a blue chip, a white chip, and a black chip.

To play this game you pick two of the chips from the box at the same time without looking.

You win if you pick the two chips which are colored red and white.

You lose if you do not pick these two chips.

Table 12
Data for Item 7

LEVELS OF STIMULI

IQ	V			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	0/3	1/2	0/3	1/3	0/1	0/3	1/3	1/2	0/3
School A (Grade 6)	0/2	0/3	0/3	0/2	0/3	0/3	1/2	0/3	1/3	1/2	0/3	1/3
School B (Grade 5)	0/2	0/2	0/2	1/2	0/2	0/2	0/1	1/2	0/2	1/2	1/2	0/2
School B (Grade 6)	1/3	1/3	0/2	0/2	0/2	0/2	2/2	2/2	1/2	3/3	3/3	1/2
Total	1/10	1/10	0/10	1/9	1/9	0/10	4/8	3/8	2/10	6/10	5/10	2/10

If you play this game only once, what chance do you have of winning?

(This item is designed to measure the probability of a simple event in an unordered, two-dimensional sample space.)

As with Item 7, very few subjects (12/30) were able to answer the question correctly. Similar to 7, the school differences (3/16—A; 9/14—B) were caused by B—G. 6 (7/8). The breakdown of these 12 correct responses is of interest. There was quite a difference between the Fifth and Sixth Grades for School B (2/6 versus 7/8, respectively) while in School A there was little difference (1/8 versus 2/8, respectively). (See Table 13.)

The Sixth Graders from School A gave somewhat the same type of wrong answers as the Fifth Graders (i.e., 1/4—one way to win and there are four colors or 2/4—2 colors to win and 4 colors altogether). In contrast, for the Sixth Grades in School B, only one person missed the item giving the answer 1/5 based on a mistaken total of five outcomes rather than six. With respect to IQ (6/10—H; 3/10—M; 3/10—L), the differences slightly favored H.

Summary (Items 2-8)

Looking at Items 2-8, there were overall differences in performance in favor of the Sixth Graders. (See Table 14.) Most of the differences occured at the verbal and picture level. The greater grade difference was between Fifth and Sixth Graders in School B. (See Table 15.) With respect to schools, students at School B were twice as effective as those at School A in answering questions at the Verbal and Verbal Hints level combined and at the picture level. (See Table 16.) School B was also more effective overall (49% - A; 74% - B).

There was an overall, upward trend in achievement with respect to IQ (L-47%; M-59%; H-75%). There were differences between IQ and the level at which a question was answered, particularly at the Verbal level with those from H being more successful at the V and VH stimuli levels than M which in turn was more successful than L. (See Table 17.)

THREE CONCEPTS

The data for the 13 items (Table 18) were regrouped by concept. From Table 19 one can see that the concepts had the following percentages of difficulty: Concept 1 (81%), Concept 2 (71%), Concept 3 (66%).

With respect to stimuli and Concept 1 (Table 20) the verbal stimulus was more effective for School B (29/42 or 69%) than School A (20/48 or 42%). (Note 20/48 means that

Table 13
Data for Item 8

LEVELS OF STIMULI

IQ	V			P			M			Total		
	H	M	L	H	M	L	H	M	L	H	M	L
School A (Grade 5)	0/3	0/2	0/3	0/3	0/2	0/3	1/3	0/2	0/3	1/3	0/2	0/3
School A (Grade 6)	0/2	0/3	0/3	0/2	0/3	1/3	1/2	0/3	0/2	1/2	0/3	1/3
School B (Grade 5)	0/2	1/2	0/2	0/2	0/1	0/2	1/2	0/1	0/2	1/2	1/2	0/2
School B (Grade 6)	2/3	0/3	0/2	1/1	0/3	0/2	0/0	2/3	2/2	3/3	2/3	2/2
Total	2/10	1/10	0/10	1/8	0/9	1/10	3/7	2/9	2/9	6/10	3/10	3/10

Overall Differences on Items 2-8
Terms of Percentages of Students Answering the Items Correctly

Table 14

Grade Comparison

	V + VH	P	M	Total Sum %
Fifth Grade (N=14)	23	09	20	53
Sixth Grade (N=16)	37	15	15	67

Table 15

Breakdown of Item Difficulties By Schools and Grades

	V + VH	P	M	Total Sum %
School B Fifth Grade (N=6)	29	12	17	54
School B Sixth Grade (N=8)	48	21	13	82
School A Fifth Grade (N=8)	16	05	23	44
School A Sixth Grade (N=8)	25	09	18	52

Table 16

School Comparison

	V + VH	P	M	Total Sum %
School A	21	07	21	49
School B	42	18	14	74

Table 17

IQ Comparison

	V + VH	P	M	Total Sum %
H	41	07	27	75
M	30	10	19	59
L	17	13	17	47

there were 20 correct responses out of 48 possible responses.) The picture stimulus was also more helpful for B (5/19 or 26%) than A (2/28 or 7%). The model level and verbal hints were equally helpful for A (11/26 or 42%) and B (6/14 or 43%).

The table for Concept 2 (Table 21) is presented item by item (indicating the number of correct responses) because of the dependencies of the stimulus levels for Items 1b-f. It is interesting to note the stability of the responses with regard to the stimulus levels for B in contrast to A. School B was more effective at the Verbal level than School A regardless of the item-stimulus dependencies.

For Concept 3 (Table 22) again School A (20/42 or 48%) was more successful at the Verbal level than School B (14/48 or 29%). At the picture level, A (7/34 or 21%) and B (5/22 or 23%) were approximately the same.

For all three concepts School B was more successful than School A. Concept 2 produced the largest difference (88%—A, 54%—B), and Concept 3 the smallest (71%—B, 60%—B). Concept 1 had B scoring 93% and A scoring 71%.

Hypotheses 7, 8, and 9

The ability to assign a probability to an event (Items 1(b-f), 7, 8) and the ability to select the most likely (or equally likely) event (Items 4, 5, 6) had mixed results. However there was some relationship between the five who answered Item 5 (the most difficult item on the test) correctly from School B. These students averaged 94% (33/35) on the seven items concerned with assigning a probability number to an event. (The two students from School A who got the item correct averaged 57% (4/7).) Thus the data offered weak support of Hypothesis 8.

Table 18

Item Difficulty (Ratio Form)

Item Number	A-Grade 5	A-Grade 6	B-Grade 5	B-Grade 6	Total Average
1a	8/8	8/8	6/6	8/8	30/30 (1.00)
1b	5/8	7/8	6/6	8/8	26/30 (.87)
1c	8/8	6/8	6/6	8/8	28/30 (.93)
1d	4/8	4/8	6/6	8/8	22/30 (.73)
1e	8/8	7/8	6/6	8/8	29/30 (.97)
1f	4/8	5/8	5/6	7/8	21/30 (.70)
2	4/8	5/8	5/6	7/8	21/30 (.70)
3	4/8	5/8	6/6	8/8	23/30 (.77)
4	6/8	8/8	5/6	8/8	27/30 (.90)
5	1/8	1/8	2/6	3/8	07/30 (.23)
6	7/8	6/8	5/6	7/8	25/30 (.83)
7	2/8	2/8	2/6	7/8	13/30 (.43)
8	1/8	2/8	2/6	7/8	12/30 (.40)

Table 19

Summary Table of Concepts 1, 2, and 3

	School A			School B			
	G5	G6	Total G5&G6	G5	G6	Total G5&G6	Total (A and B)
Concept 1. Points of a sample space (Items 1a, 2, 3)	16/24	18/24	34/48	16/18	23/24	39/42	73/90 (.81)
Concept 2. Probability of an event (Items 1 (b-f), 7, 8)	32/56	33/56	65/112	33/42	53/56	86/98	151/210 (.71)
Concept 3. Most likely and equally likely events (Items 4, 5, 6)	14/24	15/24	29/48	12/18	18/24	30/42	59/90 (.66)

Table 20

Concepts and Stimuli

Concept 1

	School A						School B					
	G5			G6			G5			G6		
	V + VH	P	M	V + VH	P	M	V + VH	P	M	V + VH	P	M
Concept 1 (Items 1a, 2, 3)	9/24	1/24	6/24	11/24	1/24	5/24	12/18	2/18	3/18	17/24	3/24	3/24
Grade Total	16/24			17/24			17/18			23/24		
School Total	33/48						40/42					

Table 21

Concepts and Stimuli

Concept 2

	School A						School B					
	G5			G6			G5			G6		
	V + VH	P	M	V + VH	P	M	V + VH	P	M	V + VH	P	M
1b	4	1	0	3	2	2	6	0	0	5	1	2
1c	4	1	3	3	2	1	6	0	0	5	1	2
1d	1	2	1	3	1	0	6	0	0	5	1	2
1e	0	3	5	3	1	3	6	0	0	5	1	2
1f	0	0	4	2	1	2	3	2	0	4	0	3
7	0	1	1	0	0	2	0	1	1	2	0	5
8	0	0	1	0	1	1	1	0	1	2	1	4
Subtotals	9/56	8/56	15/56	14/56	8/56	11/56	28/42	3/42	2/42	28/56	5/56	20/56
Grade Totals	32/56			33/56			33/42			53/56		
School Totals	55/112						86/98					

Table 22

Concepts and Stimuli

Concept 3

School B	G5	V + VH	7/18	12/18	30/42		
		P	2/18				
		M	3/18				
	G6	V + VH	13/24	18/24			
		P	3/24				
		M	2/24				
School A	G5	V + VH	5/24	14/24	29/48		
		P	4/24				
		M	5/24				
	G6	V + VH	9/24	15/24			
		P	3/24				
		M	3/24				
Concept 3 (Item 4, 5, 6)				Grade Total	School Total		

With regard to listing sample points (Items 1a, 2, 3) and choosing the most likely (or equally likely) event, the seven who answered Item 5 correctly had 20/21 correct responses on the three items concerned with listing sample points. It should be noted that all 7 Ss were in the H and M IQ groups (5 Ss - H, 2 Ss - M). Only 3 Ss were able to answer 4, 5, and 6 correctly.

There was an interesting relationship between the listing of sample points in a two-dimensional space and the probability of a simple event in that space. Since Items 3 and 8 have the same sample space (this is also true for Items 2 and 7) it was interesting to note that 23/30 were able to answer Item 3 correctly (listing the six sample points) while in Item 8 only 12/30 were able to quantify correctly a simple event in the same sample space. Of the 12/30 who answered Item 8 correctly, all 12 answered Item 3 correctly. Hence the ability to count the sample points in a sample space was a necessary but not sufficient skill to be mastered in order to quantify an event. (Similar results were true also for Items 2 and 7 in that 20/30 answered Item 2 correctly and 13/20 answered Item 7 correctly. Of the 13 who answered Item 7 correctly, 12/13 answered Item 2 correctly.)

Relation to Leffin's Results

Although no meaningful comparisons can be made due to the differences in procedure and the extreme differences in the samples used (Leffin's study used Ss from schools in Wausau, Wisconsin, representing Ss with rural and urban backgrounds), it is interesting to speculate why there was a consistent increase in item difficulties (Leffin's items would be at the picture level of stimulus for this study). It is the opinion of this author that the model and verbal hints helped substantially in improving Ss performance, particularly for the S from A on Items 2 and 3 and all Ss on Items 7 and 8 (particularly B, G-6). (See Table 23.)

Summary Hypotheses (1-9)

In summary of the results of Items 2-8 and the applicable hypotheses (Hypothesis 1 - Hypothesis 6), all six hypotheses are supported by the data. Thus in considering all the items [1(b-e) and (2-8)], the first six hypotheses are supported by the data. (Hypothesis 1 has very weak support from 1(b-e) with Grade 5 averaging 83% and Grade 6 averaging 85%.)

Table 23

Item Difficulties Related to Leffin's Test

Items	Results from this Study		Leffin's Results	
	Grade 5	Grade 6	Grade 5	Grade 6
2	9/14 (.57) .93*	12/16 (.75) .90*	.45 *	.56 *
3	10/14 (.71) .90*	13/16 (.81) .94*	.49 *	.67 *
4	11/14 (.79)	16/16 (1.00)	.63	.73
5	3/14 (.21)	4/16 (.25)	.15	.16
7	4/14 (.29)	9/16 (.56)	.16	.22
8	3/14 (.21)	9/16 (.56)	.02	.02

* These scores represent the item difficulties using the partial scoring procedure used by Leffin (e.g., if a S listed three of four possible sample points he receives a score of .75 on the item).

With regard to Hypotheses 7, 8, and 9, Hypothesis 7 has substantial support; Hypothesis 8, weak support; and Hypothesis 9, inconclusive support.

IMPLICATIONS

The children involved in the study were able, with a minimum amount of hints and guidance, to average 84% on Items 1-5 and 60% on Items 6-12 for an overall average of 70%. One can thus be fairly confident that a unit on probability including these simple concepts of probability would be within the mental grasp of these children. Their environment, although void of formal instruction in probability, has communicated to them some knowledge of probability.

One can also see some evidence for sequencing the instruction of these three topics. It appears that listing outcomes of a sample space should be taught first, followed by assigning probabilities to events and then followed by using these in making decisions concerning most likely events. (This is also the logical order one would normally plan to carry out instruction for these topics.)

The typical units on probability for the elementary school, in light of this study, would be appropriate for the children from School B. However, they would probably be very inadequate for the children from School A. These children seemed to have serious verbal problems and would need repetition of the verbal questions, many visual devices, such as pictures, and also models to play with. Given this type of instruction, conducted in an interesting and meaningful fashion for their particular needs, they too should be able to master these concepts.

LIMITATIONS

One should keep in mind the limited generality of this study. The sample is small and nonrandom. Due to local difficulties it was not possible to assign students randomly to interviewers. The data have been analyzed from many different viewpoints, of which some are certainly dependent and contribute to the differences cited for another factor. Part of the differences between schools may have been caused by the regression to the mean factor operating in connection with the selection of students by IQ levels. The students classified in the high IQ group from School A are certainly further from their school IQ mean than their counterparts from School B.

SUGGESTIONS FOR FURTHER STUDY

The study lends evidence to the feasibility of including these three concepts in the elementary program. A study is needed to demonstrate whether these concepts can be taught successfully to elementary students with different backgrounds.

Also needed as a follow up are carefully controlled experiments to test whether the observed trends in this study hold up in relation to different types of population. If this is done, then this study will have served its purpose as an exploratory study of the status of basic concepts in probability involving different kinds of student and different levels of stimuli.

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APPENDIX A

Introduction, Items, and Pictures Used in the Interview.

INTERVIEW

Introduction to Interview

Hello _____. I am ( ) from Madison. (Small talk to establish rapport.)

The purpose of this interview is to ask some questions which involve experiments and games.

What we are trying to do is to learn how you think about certain games and experiments. I would like you to help us in this project. This is not a test and your teacher will not be given the results.

What I would like you to do is to answer some questions about some experiments and games and for you to tell me the thoughts that come to you as you think about them. I would also like you to tell me, if you can, how you thought of your answer. I will be asking you why you gave that answer. This does not mean your answer is right or wrong, but that I am trying to find out how you arrived at your reply. Now these are the models that we are going to use. (Show Ss box with chips and spinners.)

Are there any questions before we begin?

In this experiment a box contains six chips—three white, two red, and one black. You pick <u>one chip</u> from the box without looking.

a. What are all the different colors it would be possible to obtain for this experiment?

b. What is your chance of getting a <u>white</u> chip?

c. What is your chance of getting a <u>black</u> chip?

d. What is your chance of getting a <u>red</u> or <u>white</u> chip?

e. Imagine that the first chip you pick is a black chip. <u>You do not</u> put this chip back into the box. If you pick again, now what is your chance in picking a black chip? a red or a white chip?

<u>Hint</u>

1.V What I mean is an answer like 1/5 or 3/11.
2.V For example, there are 2 red and 6 chips altogether so your chances are 2/6.
3.M What do you think of your chances of winning? (Are they good, bad, or neither?)
4.M How many chips altogether?
5.M How many (white, black, or red) chips are there?

In this experiment you have two spinners. The first spinner is colored half-white and half-red. The second spinner is colored half-blue and half-yellow.

You spin the arrow on <u>each</u> spinner. (If an arrow stops on a line you spin it again.)

Write the list of all the different sets of two colors it would be possible to obtain for this experiment. (Are these all of them or are there more?)

<u>Hint</u>

1.V For example, one set is (white, yellow).
2.M What does the word <u>each</u> mean?
3.M Are (red, blue) and (blue, red) different?
4.M Try the spinner and see what colors you get?
5.M Now without further spinning can you list any more or do you have all of them?
6.M Repeat (4).

In this experiment a box contains four chips— there is a red chip, a blue chip, a white chip, and a black chip.

This time you pick <u>two chips at a time</u>.

What are all the different sets of two chips that it would be possible to obtain? (Are these all of them or are there more?)

<u>Hint</u>

1.V For example, one set of two chips is (red, white).
2.M Are (red, blue) and (blue, red) different?
3.M Pick two chips from the box without looking and see what you get.
4.M Now without picking any more can you list any more or do you have all of them?
5. Repeat (3).

Two boxes containing black and red chips are used to play this game. The first box contains three black chips and four red chips. The second box contains three black chips and five red chips.

To play this game you pick one chip from a box without looking. You win if you pick a black chip.

If you can play this game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

<u>Hint</u>

1.M How many ways can you win in the first box? in the second box?
2.M How many ways can you lose in the first box? in the second box?
3.M What are your chances of winning in the first box? in the second box?

Two boxes containing black and red chips are used to play this game. The first box contains one black chip and two red chips. The second box contains two black chips and four red chips.

To play this game you pick one chip from a box without looking. You win if you pick a black chip.

If you can play the game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

Hint

1.M How many ways can you win in the first box? in the second box?
2.M How many ways can you lose in the first box? in the second box?
3.M What are your chances of winning in the first box? in the second box?

Two boxes containing black and red chips are used to play this game. The first box contains one black chip and three red chips. The second box contains two black chips and eight red chips.

To play this game you pick one chip from a box without looking. You win if you pick a black chip.

If you can play this game only once, which box would you choose so that you would have the best chance of winning or doesn't it make any difference?

Hint

1.M How many ways can you win in the first box? in the second box?
2.M How many ways can you lose in the first box? in the second box?
3.M What are your chances of winning in the first box? in the second box?

Two spinners are such that one is colored half-red and half-white and the other spinner is colored half-blue and half-yellow.

To play this game you spin the arrow on each of the spinners. (If an arrow stops on a line you spin it again.)

You win if one arrow points to red and the other arrow points to yellow.

You lose if the arrow stops in any other way.

If you only play this game once, what chance do you have of winning?

Hint

1.M What do you think of your chances of winning? (are they good, bad, or neither?)
2.M How many ways can you win?
3.M How many different ways can the game come out?
4.M Spin each spinner and see what you get.
5.M Repeat (3).

A box contains four chips—a red chip, a blue chip, a white chip, and a black chip.

To play this game you pick two of the chips from the box at the same time without looking.

You win if you pick the two chips which are colored red and white.

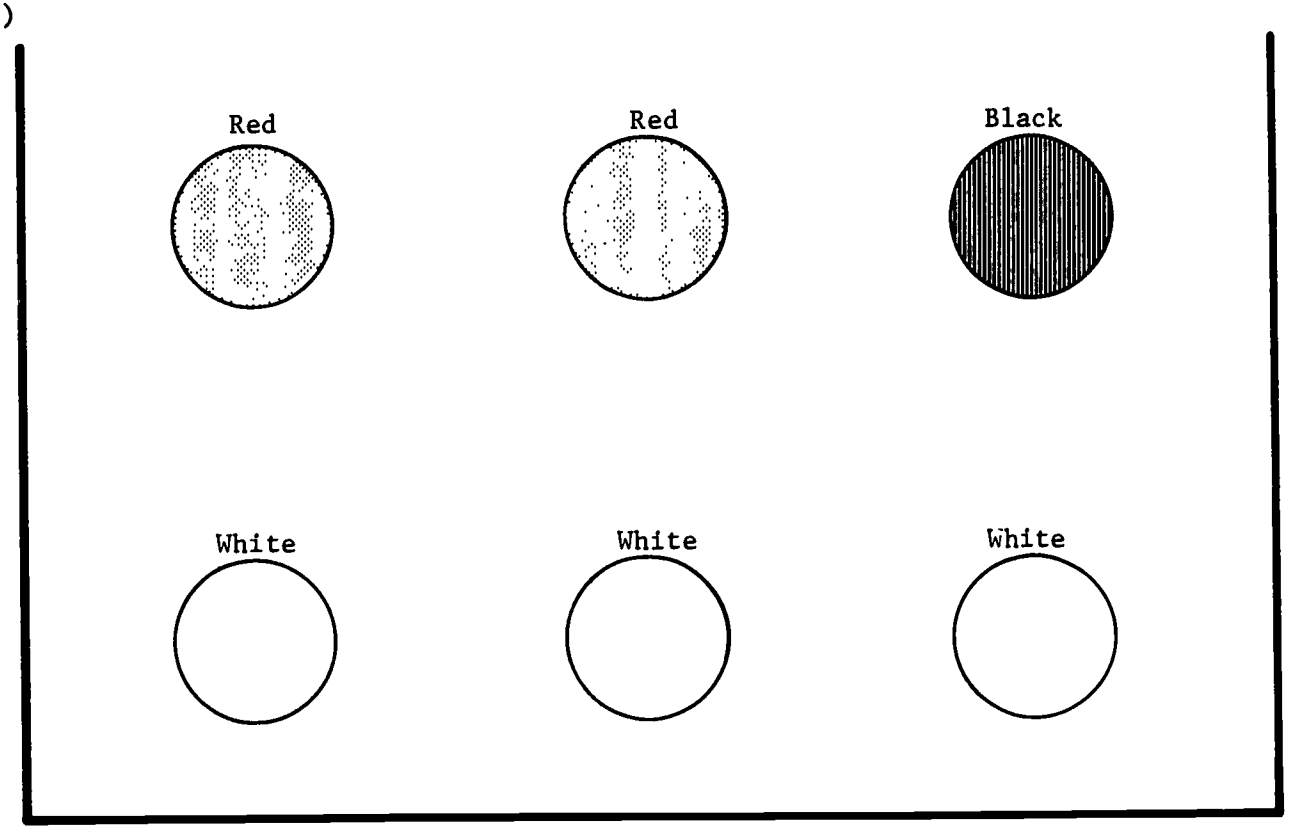
You lose if you do not pick these two chips.

If you play this game only once, what chance do you have of winning?

Hint

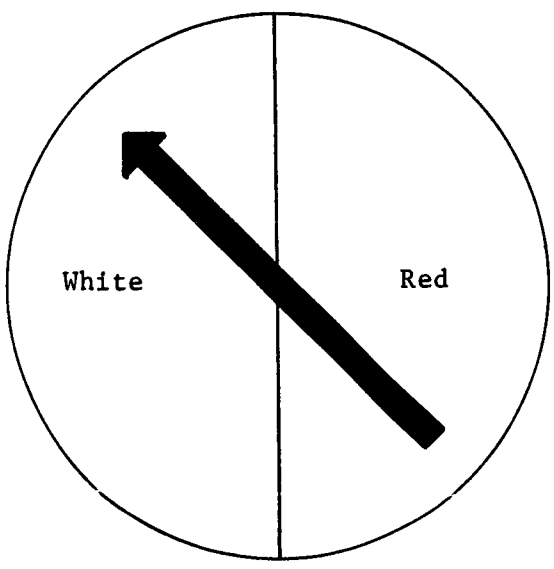
1.M What do you think of your chances of winning? (Are they good, bad, or neither?)
2.M How many ways can you win?
3.M How many different ways can the game come out?
4.M Pick two chips from the box and see what you get.
5.M Repeat (3).

1 (a - f)

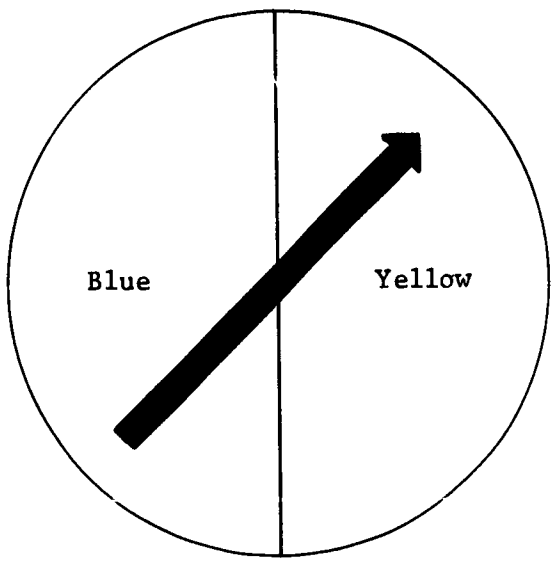


2, 7

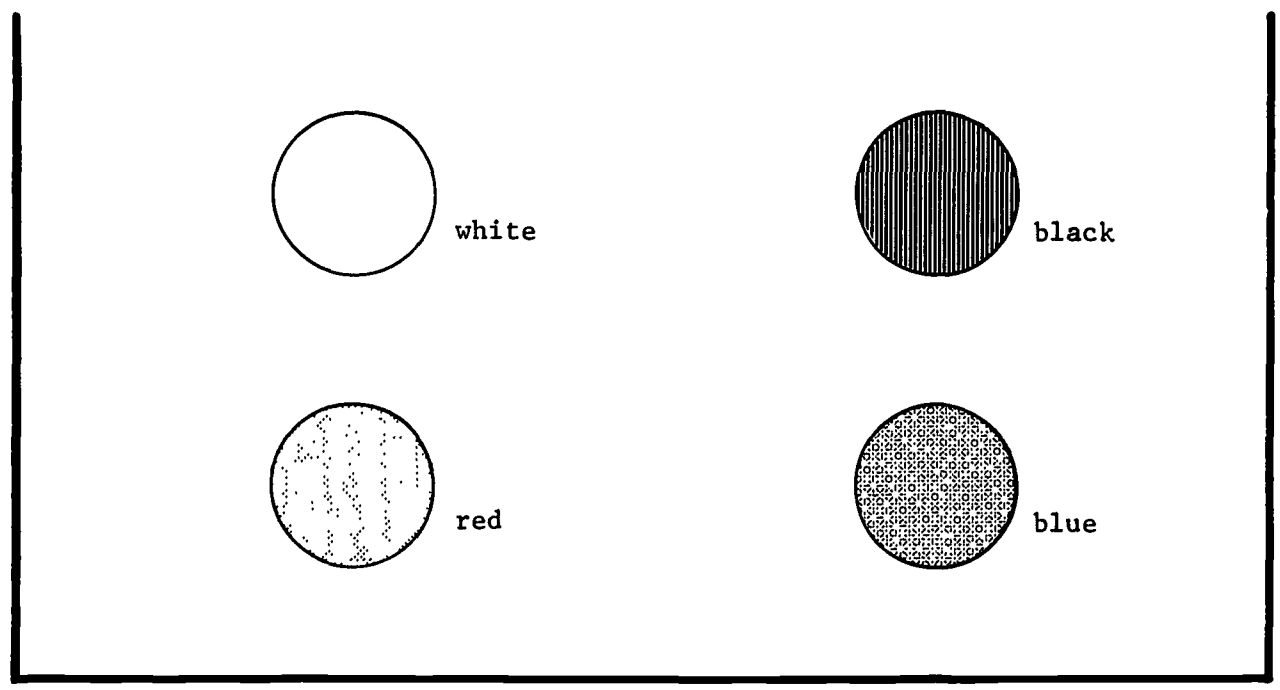
FIRST SPINNER



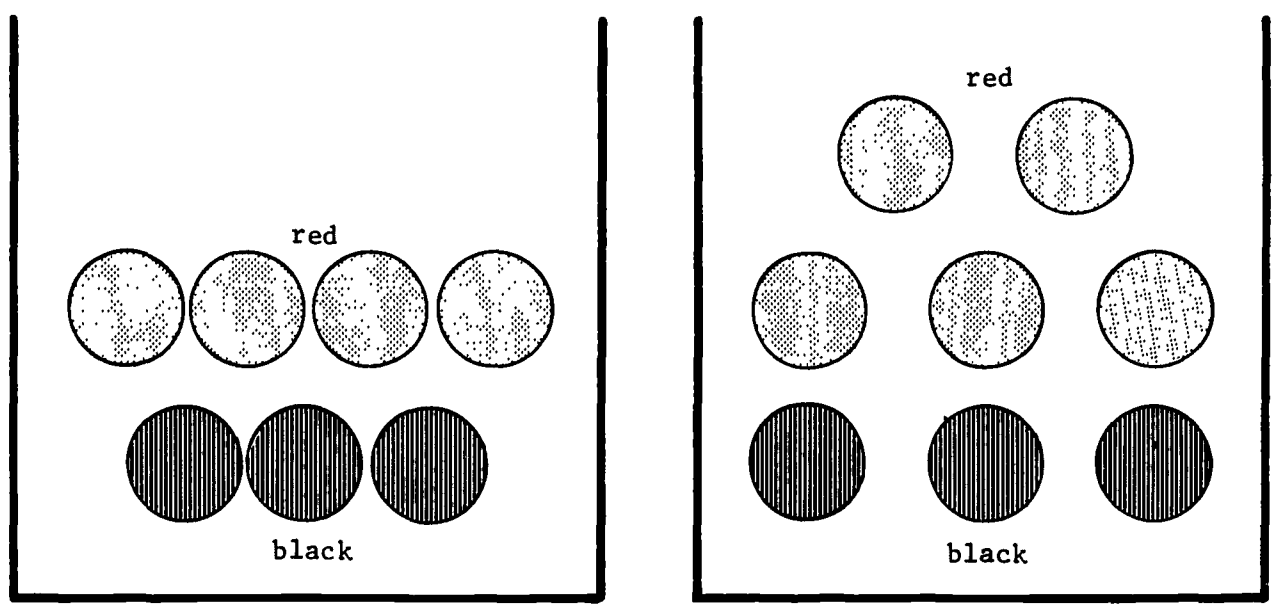
SECOND SPINNER



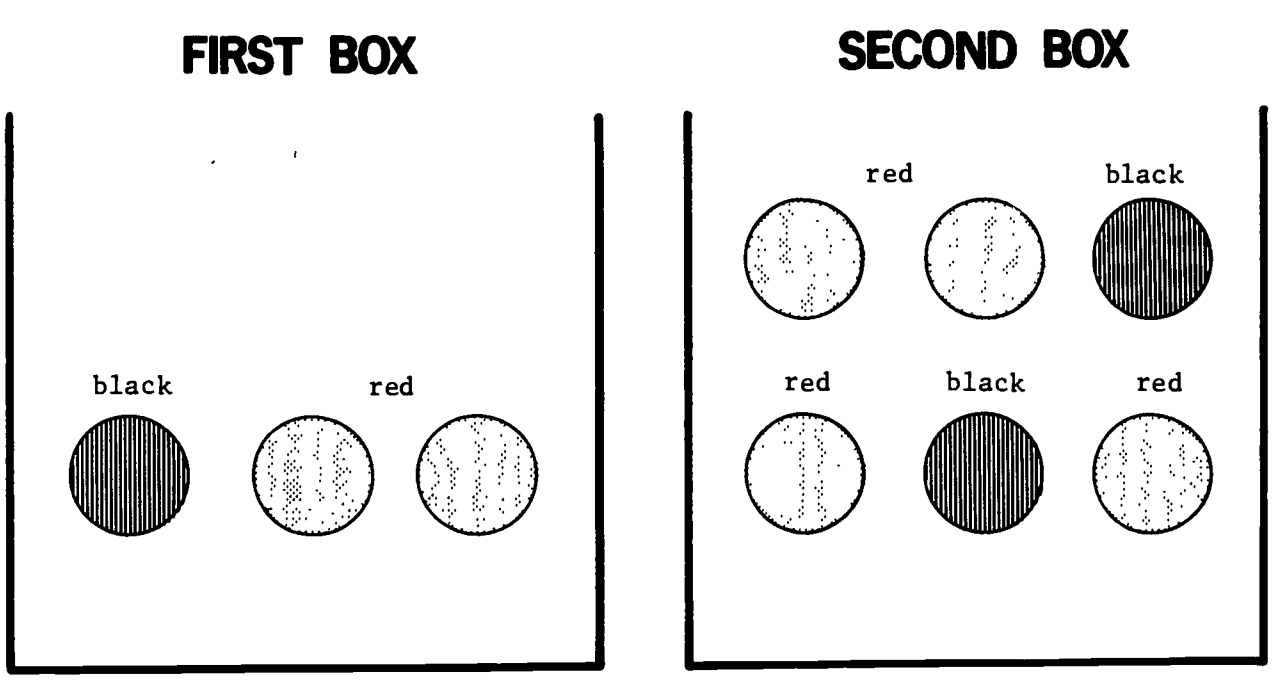
3,8


white
black
red
blue

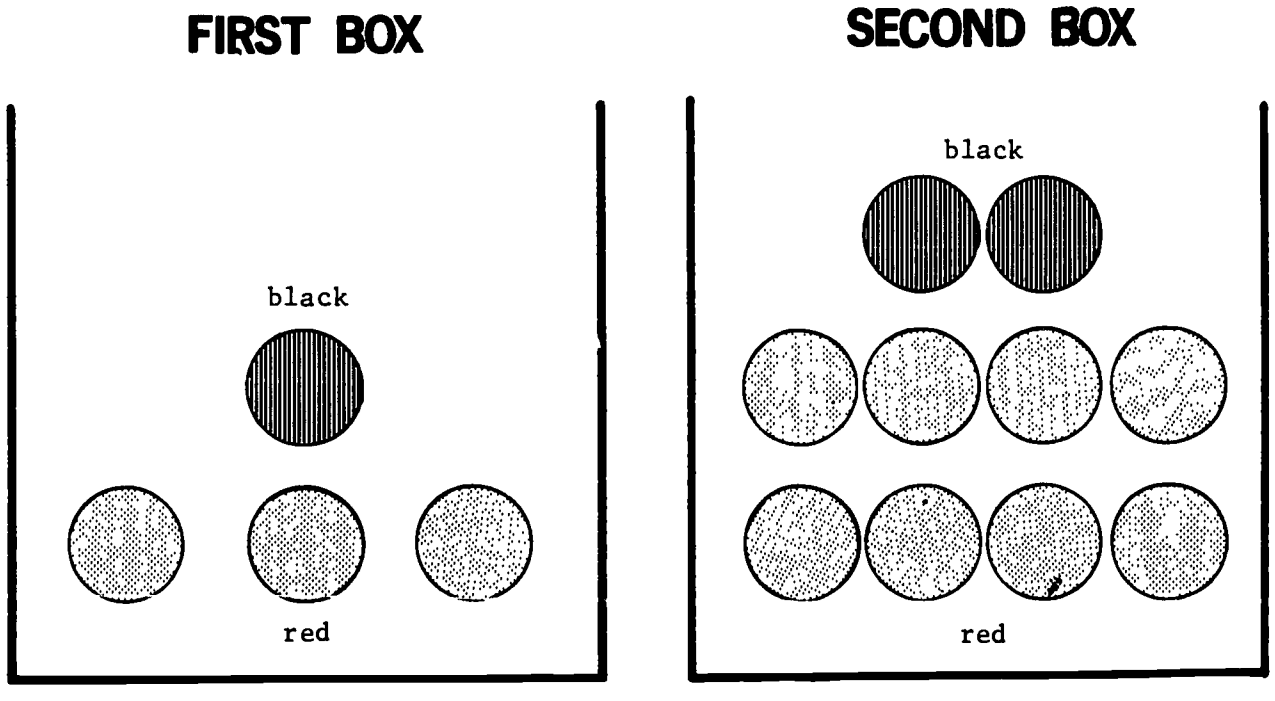
4


FIRST BOX
red
black
SECOND BOX
red
black

5


FIRST BOX
black
red
SECOND BOX
red
black
red
black
red

6


FIRST BOX
black
red
SECOND BOX
black
red

APPENDIX B

Recording Sheets for Interviewer and Interviewee

GPO 817-178 - 3

Recording Sheet Used by Interviewer

NAME _____

M.I.Q.S. _____

GRADE _____ 5 _____ 6 _____ 7.

	V (No H)			V H			P			M			
Question	R	W	Reason	R	W	Reason	R	W	Reason	R	W	H.Used	Reason
1. a													
b			A G U M	1 2		A G U M			A G U M			1 2 3 4 5	A G U M
c			A G U M	1 2		A G U M			A G U M			1 2 3 4 5	A G U M
d			A G U M	1 2		A G U M			A G U M			1 2 3 4 5	A G U M
e			A G U M	1 2		A G U M			A G U M			1 2 3 4 5	A G U M
f			A G U M	1 2		A G U M			A G U M			1 2 3 4 5	A G U M
2.	/4	/4		/4	/4		/4	/4		/4	/4	1 2 3 4 5	
3.	/6	/6		/6	/6		/6	/6		/6	/6	1 2 3 4 5	
4.			B, U			B, U			B, U			1 2 3	B, U
5.			C D E U			C D E U			C D E U			1 2 3	C D E U
6.			D E F U			D E F U			D E F U			1 2 3	D E F U
7.			A G U M			A G U M			A G U M			1 2 3 4 5	A G U M
8.			A G U M			A G U M			A G U M			1 2 3 4 5	A G U M

Sheet Used to Help Interviewer in Recording Reasons

Typical Responses for Question, "Why?"

Response applicable to:	
1, 7, 8	A. There are (is) _____ favorable and _____ altogether.
4	B. The first box since there are the same number of black chips in both boxes and there are less red chips in the first box (or more red chips in the second box).
5	C. They are the same since 1 out of 3 is the same as 2 out of 6.
5, 6	D. The second box, since there are more black chips in the second box than there are in the first box.
5, 6	E. The first box, since the second box has more red chips.
6	F. The first box since 2 out of 5 is better than 3 out of 9.
1, 7, 8	G. I think _____ out of _____ (Guess).
1, 7, 8	M. _____ out of _____ since I have gotten _____ winners out of _____ trys.
1, 4, 5, 6, 7, 8	U. Unusual response (Record tape number).

Student Response Sheet

NAME _____

GRADE _____

1. a
   b
   c
   d
   e
   f

2.

3.

4.

5.

6.

7.

8.

GPO 817-179-2